

## Problem Corner

Solutions are invited to the following problems. They should be addressed to **Nick Lord** at **Tonbridge School, Tonbridge, Kent TN9 1JP** (e-mail: [njl@tonbridge-school.org](mailto:njl@tonbridge-school.org)) and should arrive not later than 10 March 2020.

Proposals for problems are equally welcome. They should also be sent to Nick Lord at the above address and should be accompanied by solutions and any relevant background information.

### 103.I (Martin Lukarevski)

Let  $x, y, z$  be positive real numbers such that  $x + y + z = xyz$ . Prove that

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} + \frac{8}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}} \geq 2.$$

When does equality hold?

### 103.J (Joseph Tonien)

Find all functions  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  such that:

- $f(2019) = 2018$ ;
- $f(n - f(n)) = 0$  for all  $n \in \mathbb{Z}$ ;
- there is an integer  $t$  such that, for any fixed  $n \in \mathbb{Z}$ , the equation  $f(x) = n$  has exactly  $t$  solutions for  $x$ .

### 103.K (Yagub N. Aliyev)

Given triangles  $ABC$  and  $A_1B_1C_1$ , denote by  $K, L$  and  $M$  the intersection points of  $BC_1$  and  $B_1C$ ,  $CA_1$  and  $C_1A$ ,  $AB_1$  and  $A_1B$  respectively. Prove that the lines  $AA_1, BB_1$  and  $CC_1$  are concurrent if, and only if, the intersection points of  $BC$  and  $LM$ ,  $CA$  and  $MK$ ,  $AB$  and  $KL$  are collinear.

### 103.L (Tony Crilly and Stan Dolan)

Let  $a, b$  and  $c$  be positive integers such that an  $a \times a$  square can strictly contain a  $b \times c$  rectangular hole.

Find, with proof, all possibilities for which the area of this *Emmental square* is numerically equal to its perimeter.

For example, the Emmental square in the figure with a square hole has area  $36 - 4 = 32$  and perimeter  $4 \times 6 + 4 \times 2 = 32$ .

